Local Financing of Continuous Coastal Protection under Fragmented Ownership[1]

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Abstract Coastal protection against sea-level rise and storm surge may be planned as one continuous line but delivered by separate landowners. Even with positive aggregate private surplus, the line can remain unfinanceable when parcels with available participation margin are ineligible to fund a costly interface. This paper characterises that constraint under a fixed assessment rule. Parcels form a path, receive private value only when the full line is documented as complete, and may finance an interface only within a specified payer radius. A radius-specific allocation supports full completion as a Nash equilibrium if and only if every parcel has a non-negative participation margin and every contiguous block of interfaces costs no more than the participation margin in its expanded payer neighbourhood. The condition identifies the maximum block deficit and the minimum feasible payer radius. The model distinguishes the problems of a negative parcel margin, an aggregate private deficit, a spatially inaccessible margin, and equilibrium selection. Signed transfers address the first; outside finance covers the second when external value makes completion socially worthwhile; broader incidence addresses the third. Commitment, not transfers, addresses the fourth: completion-contingent transfers do not remove the no-build equilibrium unless commitment changes the payoff from non-participation. Singapore, a low-lying island state building continuous coastal defences across separately owned land, provides the physical and institutional setting.

JEL classification: Q54, Q58, H41, C72, R52

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[1]Refine.ink was used to check the paper for consistency and clarity.

1 Introduction

Coastal protection against sea-level rise and storm surge is often planned as one system and delivered in pieces. A seawall segment, raised platform, flood gate, or tie-in belongs to a site, but the prescribed standard may require all pieces to form a continuous line. Decisions are made parcel by parcel; compliance is judged along the boundary. Fragmented ownership therefore creates two constraints. Every required parcel must participate, and the resource cost at each interface must be assigned.

This paper characterises feasible cost incidence on a path of N parcels. Incidence here is statutory. An assessment rule assigns each interface cost to named parcels, and there is no shifting of that charge through markets. Each parcel has an expected incremental value conditional on verified completion; that value already incorporates uncertain performance and residual risk. Each parcel funds its own segment. An interface may be charged only to its two endpoints or, under broader incidence, to parcels within a wider payer radius. The assessment rule is fixed before parcels choose whether to participate, and a parcel that participates cannot escape its share; the game leaves participation itself uncompelled, as a benchmark for whether delivery is privately acceptable without enforcement. The rule is not derived from bargaining.

The main result is a family of spatial budget conditions. At any payer radius, every contiguous block of interfaces must cost no more than the participation margin (each parcel's value net of its own segment cost) available in its payer neighbourhood. With four parcels, margins $(4, 0, 0, 4)$, and interface costs $(1, 2, 1)$, total margin exceeds total interface cost, yet endpoint incidence cannot finance the middle join because the margin sits two parcels away from it (Figure 1). Expanding the payer neighbourhood relaxes the conditions monotonically and identifies the minimum payer radius required for finance.

A line can fail to be financed for four distinct reasons. A parcel may have a negative margin even when the group can pay. The group itself may have an accounting deficit. Aggregate margin may be sufficient but lie outside the eligible payer neighbourhood of a costly interface. Finally, a feasible allocation may coexist with no-build. These problems call, respectively, for redistribution, outside finance, broader incidence, and commitment.

The instruments are ranked by what a regulator must be able to do rather than by what each can achieve: assessments within a payer radius move no money between owners; signed transfers require a clearing mechanism; outside finance requires an appropriation. Unrestricted transfers would also repair a wrong-location failure, so the minimum payer radius is the answer when transfers between owners are unavailable.

Private and social value are also distinct. Parcel values determine participation; benefits or harms outside the parcel group affect social welfare but not private payoffs. A socially worthwhile line may then require outside finance. Signed transfers, which may take from some parcels and pay others, can redistribute participation margin, but completion-contingent transfers alone do not remove the no-build equilibrium.

Singapore's 2026 *Code of Practice on Coastal Protection* (CPCOP) provides the institutional motivation. It is a case in which a continuous line is mandated by statute yet delivered by separate private owners, with documented interface and completion requirements (Public

Utilities Board 2026a). The Absolute Protection Boundary demarcates the protected region; measures along it must link up to form a continuous line of defence. The relevant analogue is a contiguous sequence of adjoining prescribed places serving one protected region, not the entire national coastline (Ministry of Sustainability and the Environment 2026a). Applicable normative requirements are mandatory once an owner's obligation takes effect. The game instead asks whether delivery would be privately acceptable without enforcement or non-compliance penalties, so no-build is a benchmark rather than a prediction of statutory compliance.

Section 2 positions the result. Section 3 sets out the model and its correspondence to the Code. Section 4 derives the financing-scope conditions and Section 5 the transfer, outside-finance, and commitment results. Section 6 illustrates the four cases numerically, and Section 7 collects the model's scope and extensions.

2 Relation to the literature

The binary line-completion technology is a strict weakest-link or unanimity public good. Weakest-link provision, multiple equilibria, transfers, and institution formation are established results (Hirshleifer 1983; Vicary 1990; Cornes 1993; Vicary and Sandler 2002; Caparrós and Finus 2020; Caparrós et al. 2025). Assurance contracts address non-provision equilibria in threshold environmental goods (Li et al. 2021). Threshold uncertainty can itself alter contributions and coordination (Carlsson et al. 2024). Uncertainty enters in this paper via expected parcel values while documented completion remains binary.

Network and local public-good models study how contribution links, cost shares, or group membership are formed. Sun (2017) develops Lindahl finance in networks, and Hoelzemann and Li (2026) show that acyclic local payoff networks can support a unique maximal-action equilibrium. Completion remains globally unanimous here. The network limits who may pay, not whose action determines value. In local public-good economies, Chan and van den Nouweland (2025) extend ratio equilibrium to agents who sort into jurisdictions with endogenous cost shares, and Chan and van den Nouweland (2026) establish existence and characterise that equilibrium. This paper fixes both jurisdiction and cost shares and asks whether the resulting interface obligations fit the eligible payers' participation budgets.

The underlying feasibility problem is classical. Gale's flow theorem generalises Hall's condition to capacitated networks, and convex bipartite graphs have interval neighbourhoods on one side (Gale 1957; Katriel 2008). The coastal path supplies that convexity and allows the full family of capacity tests to be written as contiguous-block inequalities. The paper specialises this classical result to the payer graph induced by a coastal path; it does not claim a new matching theorem.

The literature on spatial environmental incentives asks the most similar question, namely how to secure contiguous action across private boundaries. Agglomeration bonuses reward contiguous conservation across private boundaries (Parkhurst et al. 2002), and recent work distinguishes internal from external landholding boundaries (Bareille and Soubeyran 2025). Lange and Romero-Fernández (2024) study heterogeneous endowments and local in-kind transfers in overlapping weakest-link neighbourhoods. Cowen and Delmotte (2020) examine flood governance when property rights do not match the physical resource. In this paper, the line, the ownership map, and the interface costs are all given; the only spatial choice left is how far the assessment rule may reach to collect an interface cost.

Coastal economics studies cooperative erosion protection among private owners, decentralised beach nourishment across adjacent communities, local tax support for shoreline investment, and non-cooperative levee systems (Kriesel and Randall 1994; Hui et al. 2016; Gopalakrishnan et al. 2017; Mullin et al. 2019). Related work examines seawall bargaining, cooperation among jurisdictions protecting transport networks, compensation among upstream and downstream landowners, and Shapley cost allocation (Delille and Péreau 2014; Jeong et al. 2018; Álvarez et al. 2019; Papakonstantinou et al. 2019). Private shoreline armouring adds physical spillovers (Beasley and Dundas 2021), while a recent Singapore choice experiment finds that coastal-protection values vary with risk attitudes, trust, and proximity (Trinh et al. 2026). These studies choose investment, cooperation, taxes, subsidies, or compensation. In their settings the deliverer is a community, district, or authority, and continuity along a stretch of coastline is either not required or is that body's own responsibility. In this paper, the deliverer is the parcel and continuity is mandated, so the question is whether a given assessment rule lets enough margin be charged against each join. A block-by-block test locates the interfaces a rule cannot finance and gives the least widening of the rule that restores feasibility.

3 Model and institutional correspondence

Delivering a prescribed line of protection has four stages. A planner prescribes a design standard; an authority records documented regulatory completion against that standard; realised hydraulic performance remains uncertain because hazards may exceed the design basis and structures or operations may fail; and households and firms may take further protective action under residual risk. These are stages of delivery, distinct from the four financing failures of the introduction. The model below formalises the second stage, documented completion; uncertain performance enters parcel values in reduced form, and later protective behaviour lies outside it.

Let $N \geq 2$ parcels, indexed $i = 1, ..., N$, form a path. Interface $e \in \{1, ..., N-1\}$ joins parcels e and $e+1$. Parcel i chooses $y_i \in \{0, 1\}$. Here $y_i = 1$ means choosing high and, by assumption, delivering a segment subsequently verified against the prescribed standard; $y_i = 0$ means not doing so. Verified completion is

$$C(y) = \prod_{i=1}^{N} y_i. \tag{1}$$

The product equals one only when every parcel builds, because a flood defence with a gap protects little of the region behind it.

Parcel i has private value $b_i \geq 0$ conditional on completion and pays segment cost $c_i > 0$ when it chooses high. Agents are risk neutral with quasilinear expected monetary payoffs. Net external value to parties outside the parcel group is $g \in \mathbb{R}$. Interface e has divisible resource cost $\kappa_e \geq 0$. Joins are costly when adjoining structures differ: a wall meeting a raised platform or gate needs a bespoke connection, a structure tied into an older neighbour needs survey and strengthening work, and two parcels building to different protection levels under the lease rule of Section 3.1 need a transition between them. Such a join can cost more than either adjoining parcel's margin can bear, which is the case the examples below isolate.

A payer radius $h \in \{0, ..., N-2\}$ permits interface e to be charged to

$$P_e(h) = \{i : \max(1, e-h) \leq i \leq \min(N, e+1+h)\}. \tag{2}$$

The radius counts steps on the parcel path, not physical distance or administrative boundaries. Thus $h = 0$ restricts payment to the two adjoining parcels, while $h = N-2$ allows every parcel on the line to be charged for any interface. A cost allocation has $x_{ie} \geq 0$, $x_{ie} = 0$ outside $P_e(h)$, and

$$\sum_{i \in P_e(h)} x_{ie} = \kappa_e. \tag{3}$$

Treat x as a prospective assessment for non-refundable design, tie-in, or construction commitments. Liability is triggered by participation: a parcel that chooses high cannot avoid its shares, but a parcel that chooses low owes nothing. Parcel i incurs its shares when it chooses high, whether or not other parcels follow; expenditure is sunk once participation begins. This keeps the participation decision itself unenforced, which is the benchmark the introduction describes.

Let $t_i \in \mathbb{R}$ be a transfer paid only on verified completion. Given x and t, payoff is

$$u_i(y; x, t) = (b_i + t_i)C(y) - \left(c_i + \sum_e x_{ie}\right) y_i. \tag{4}$$

Social welfare is the sum of parcel payoffs, net of transfers, plus external value:

$$W(y; x) = \left(\sum_i b_i + g\right) C(y) - \sum_i \left(c_i + \sum_e x_{ie}\right) y_i. \tag{5}$$

At full completion, this becomes $\sum_i b_i + g - \sum_i c_i - \sum_e \kappa_e$. Every non-zero incomplete profile incurs positive cost without completion value; no-build has zero welfare.

Two benchmarks anchor the financing results. No-build is always an equilibrium, and when interfaces cost nothing each parcel's decision depends only on its own benefit and cost. Write the participation margin as $a_i = b_i - c_i$, the most a parcel can contribute to interface costs without preferring to withdraw. The margin is a parcel-level quantity measured before interface charges; private surplus is the group total $\sum_i a_i - \sum_e \kappa_e$ after them.

Proposition 1 (benchmark): No-build is a strict Nash equilibrium. With no interface costs or transfers, full completion is a Nash equilibrium if and only if $a_i \geq 0$ for every parcel. It is socially efficient if and only if $\sum_i b_i + g \geq \sum_i c_i$.

3.1 Institutional correspondence

The CPCOP assigns interface design and inspection to adjoining owners. The Professional Engineer for the preceding measure submits a Development Interface Report; the engineer for the succeeding measure designs the joint from it; the two owners inspect key construction stages together, or agree an integrated design and record who builds, inspects, and maintains it; otherwise maintenance follows plot boundaries (Public Utilities Board 2026a). This interface regime assigns design and inspection duties, not interface cost, so the payer radius h is a modelling object rather than a provision of the Code. The Government has also announced a capital grant for qualifying studies, service diversion, and construction, paid in reimbursement tranches (Ministry of Sustainability and the Environment 2026b, a). Its published terms identify neither an eligible-payer radius nor a mapping to f, t_i, or resource costs.

The Code's lease rule supplies one benefit-side comparative static. A development with more than 30 years of lease remaining at the second notification may build first to the Coastal Protection Level for the 0.7 m sea-level-rise scenario and later to the 2.15 m level, or once to the 2.15 m level; one with 30 years or less must build no lower than the 0.7 m level (Public Utilities Board 2026a). Notification leaves at least five years for implementation, so the 2028 commencement is not a common construction date (Ministry of Sustainability and the Environment 2026a; Public Utilities Board 2026b). If parcel i values a non-negative protection flow $v_i(t) \geq 0$ only until T_i, with discount function $D_i(t) \geq 0$, then $b_i(T_i) = \int_0^{T_i} D_i(t) v_i(t)\, dt$. Holding c_i and κ_e fixed, a shorter horizon weakly lowers a_i; while the existence conditions of Section 4 continue to hold it weakly raises the minimum payer radius, and otherwise it weakly raises the outside finance f^* of Section 5. The lease rule also changes present segment and interface costs, so the total effect is unsigned.

4 Financing scope

Bounded payer radii are the realistic default for three reasons the model does not formalise. Assessments for local improvements ordinarily require a nexus between payer and work, and legal standing weakens as the join recedes from the payer's own structure. Adjoining owners are already identified in the interface report and inspect the work jointly; a regional levy would need its own apportionment and collection apparatus. A charge on distant owners for a join they never touch invites objection in proportion to distance. The Code's interface regime, which places design and inspection duties on adjoining owners, is a radius-zero rule for duties; the results below ask what radius a cost rule in the same register would need. Widening the radius therefore carries rising legal, administrative, and political cost, which is why the least sufficient radius is the object of interest.

Full completion is a Nash equilibrium under an allocation x and zero transfers if and only if $\sum_e x_{ie} \leq a_i$ for every parcel.

Proposition 2 (financing within a payer radius): A radius-h allocation that makes full completion a Nash equilibrium exists if and only if $a_i \geq 0$ for every parcel and, for every contiguous block of interfaces $r, \ldots, s$,

$$\sum_{e=r}^{s} \kappa_e \leq \sum_{i=\max(1,r-h)}^{\min(N,s+1+h)} a_i. \tag{6}$$

For each block, define its radius-h financing deficit as

$$D_h(r,s) = \sum_{e=r}^{s} \kappa_e - \sum_{i=\max(1,r-h)}^{\min(N,s+1+h)} a_i, \qquad \Delta_h = \max_{1 \leq r \leq s \leq N-1} D_h(r,s). \tag{7}$$

When every $a_i \geq 0$, a positive $D_h(r,s)$ is the amount by which that block exceeds the participation margin legally available to it. A maximising block is a *witness*. It certifies that no radius-h allocation exists and shows where the payer restriction binds.

The full-completion equilibrium may be weak when a participation or block condition binds. A radius-h allocation that makes full completion a strict Nash equilibrium exists if and only if every $a_i > 0$ and every inequality in Equation 6 is strict.

At $h = 0$, each block must be financed by its incident parcels. Aggregate private surplus is not sufficient. For example, let $a = (4, 0, 0, 4)$ and $\kappa = (1, 2, 1)$. Private surplus after interface costs is four, but the middle interface has no endpoint margin: $D_0(2,2) = 2$. It becomes feasible at $h = 1$, when the outer parcels may share that liability. Figure 1 shows the two payer sets.

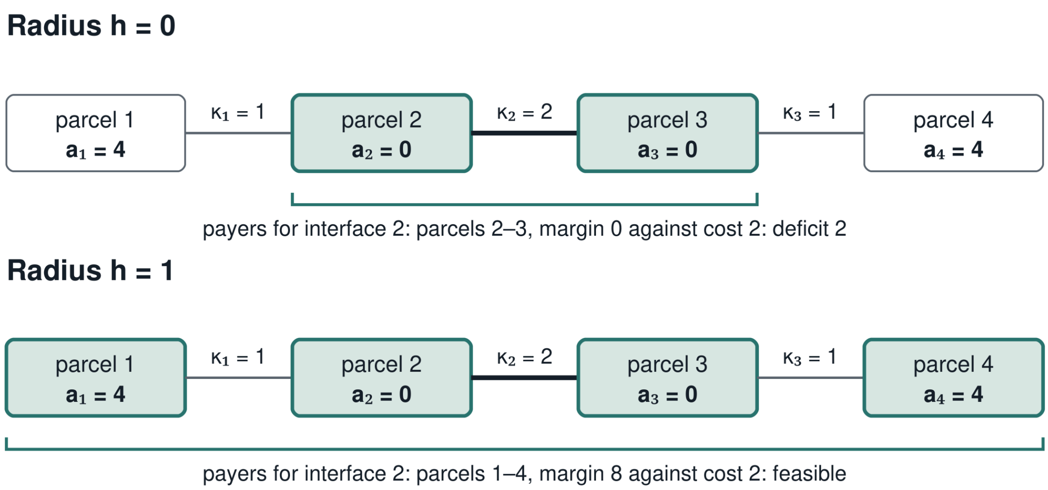

Figure 1: Payer sets for the middle interface at radius 0 and radius 1 in the four-parcel example; see Proposition 2.

The scope conditions are nested. If finance is feasible at h, it is feasible at every larger radius. If every $a_i \geq 0$, the minimum feasible radius is

$$h^* = \min\{h : \Delta_h \leq 0\}, \tag{8}$$

whenever this set is non-empty.

Proposition 3 (minimum payer radius): The minimum radius h^* exists if and only if every $a_i \geq 0$ and $\sum_i a_i \geq \sum_e \kappa_e$.

The result identifies when broader cost incidence is necessary; what institution should hold that incidence is left open (Section 7).

5 Private finance, social value, and commitment

Section 4 held to the regime in which money moves only from owners to interface costs. Broader non-negative cost incidence cannot compensate a parcel with $a_i < 0$, and no radius can cover an aggregate shortfall. Now allow signed completion-contingent transfers across the boundary, and let their net receipt from outside the parcel group be $f \geq 0$, so $\sum_i t_i = f$.

Proposition 4 (transfers and outside finance): For any fixed allocation of real interface costs, transfers that make full completion a Nash equilibrium exist if and only if

$$\sum_i b_i + f \geq \sum_i c_i + \sum_e \kappa_e. \tag{9}$$

The minimum accounting contribution from outside the parcel group is

$$f^* = \max\left\{0, \sum_i c_i + \sum_e \kappa_e - \sum_i b_i\right\}. \tag{10}$$

Balanced transfers within the boundary are the case $f = 0$. They replace spatial and individual participation constraints with the parcel group's aggregate condition. Social efficiency instead requires

$$\sum_i b_i + g \geq \sum_i c_i + \sum_e \kappa_e. \tag{11}$$

External value can make a line socially worthwhile even when $f^* > 0$. The quantity f^* is an accounting deficit, not an optimal subsidy. It treats outside finance as a transfer and omits collection, administration, and distortionary financing costs.

Because transfers are paid only on completion, they cannot remove the no-build equilibrium. At no-build, a unilateral high action incurs cost without completing the line, so no-build remains a Nash equilibrium (Proposition 1). A mandate, binding collective contract, or non-performance assessment can change that payoff. Assurance payments may also alter non-provision incentives, but their design is a separate mechanism problem (Li et al. 2021).

6 Numerical illustration

Table 1 shows why total surplus does not identify the appropriate instrument. The first two rows isolate location. Endpoint incidence is strictly feasible in the first, while the second has $\Delta_0 = 2$ despite positive private surplus and requires $h^* = 1$. The remaining rows fail before the spatial test. One has a negative parcel margin, and the other has an aggregate private deficit despite positive social surplus.

Table 1: Endpoint diagnostic and required instrument in four cases (signed Δ_0 for non-negative margins; otherwise the largest parcel deficit).

Case and parameters	Private / social surplus	Binding block or parcel	Δ_0 or parcel deficit	Result
Endpoint: $a = (2, 1, 2, 1)$; $\kappa = (1.5, 1, 1.5)$	2 / 2	interface 1	−1.5	$h^* = 0$
Wider scope: $a = (4, 0, 0, 4)$; $\kappa = (1, 2, 1)$	4 / 4	interface 2	2	$h^* = 1$
Negative margin: $a = (-1, 5, 0)$; $\kappa = (1, 1)$	2 / 2	parcel 1	1	Signed transfer
External gap: $a = (0, 0, 0)$; $\kappa = (1, 1)$; $g = 3$	−2 / 1	interfaces 1-2	2	Outside finance $f^* = 2$

Implementation begins by defining the prescribed line and the evidence required for completion, then estimating private values, external effects, segment costs, and interface costs. Test endpoint incidence and report Δ_0 and its witness block. A positive value with adequate aggregate margin identifies a wrong-location failure, and the sequence of Δ_h values shows how far incidence must expand if transfers between owners are unavailable; where they are available, the same failure can be met by redistribution instead. A negative parcel margin calls for compensation; an aggregate private deficit with positive social surplus calls for outside finance and an explicit fiscal-cost calculation. Eliminating rather than coexisting with no-build requires a separate commitment instrument.

7 Scope and extensions

These results require known values, enforceable non-refundable assessments, divisible interface costs, and a fixed path. The paper does not model private information, bargaining over x, endogenous payer sets, probabilistic network failure, hydraulic compartments, or cycles and branching networks. If assessments are refunded after failed provision, the full-completion conditions are unchanged, but off-equilibrium welfare and the non-provision equilibria must be re-derived.

The block conditions apply equally to a path of shoreline stretches or neighbouring jurisdictions when the same completion technology and liability restrictions hold. The model does not choose an authority, define a political jurisdiction, or value administration, and it does not treat parcels, local authorities, states, and countries as institutionally equivalent. Comparisons across geographic scales require an explicit mapping from the path to physical distance and legal institutions.

The full-completion condition can accommodate a positive net payoff d_i from withdrawing while all others build by replacing a_i with $b_i - c_i - d_i$. The interval theorem then applies to these effective margins. If a segment has value even when the line is incomplete, the no-build result and the welfare ranking of partial lines must be re-derived; a parcel may then build alone, and a partial line may have positive welfare.

The Singapore material is institutional motivation rather than a calibrated incidence application. Public documents do not specify the assessment rule needed to recover an actual h or Δ_h, and they provide no estimates of b_i, c_i, κ_e, g, bargaining, compliance, or realised performance. An empirical application would require those inputs, include the fiscal cost of any outside finance in its welfare comparison, and report residual risk conditional on documented completion.

8 Conclusion

A total budget does not determine whether a local assessment rule can finance a continuous line of protection against sea-level rise and storm surge. The maximum block deficit locates the interfaces the rule cannot reach, and h^* gives the least payer reach that removes the obstruction. Parcel and aggregate deficits remain separate from this geographic constraint, as does equilibrium selection.

The result therefore separates assessment design from transfer and commitment design. Where the path and liability assumptions of Section 7 hold, the block-deficit test tells an aggregate shortfall apart from margin that exists but lies outside the legal payer set, and it can be run before a payer rule is fixed rather than after it fails. A boundary-wide budget is not enough; the assessment rule must reach far enough along the coast to pay for the joins between neighbours' defences that constrain it, because the sea finds the join that is left open.

Statements and declarations

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Data and code availability. An anonymised replication archive accompanies the submission. Its standard-library Python script writes the numerical tables and checks the interval characterisation against a max-flow implementation.

Generative AI use. During manuscript preparation the author used Anthropic Claude to assist with drafting, coding, structural revision, and language editing, and Refine.ink to identify errors and inconsistencies in the text. The propositions and proofs are the author's own; the author verified all code and numerical output. The author reviewed and edited all content and takes full responsibility for the manuscript.

Appendix A: Proofs

Proof of Proposition 1

If all other parcels choose zero, a unilateral high action cannot complete the line and incurs a positive segment cost. No-build is therefore a strict Nash equilibrium. If all others choose one and interface costs are absent, parcel i obtains $b_i - c_i$ by staying high and zero by withdrawing. This proves the private condition. Every non-zero incomplete profile has negative welfare, while no-build has zero welfare. The planner therefore compares full-completion welfare $\sum_i b_i + g - \sum_i c_i$ with the zero baseline.

Proof of Proposition 2

Necessity follows by summing parcel capacity constraints over the payer neighbourhood of any contiguous block. The interfaces in the block must be financed within that neighbourhood, giving Equation 6. Non-negative interface charges cannot repair $a_i < 0$.

For sufficiency, construct a flow network. The source has an arc of capacity κ_e to interface node e. Interface node e has unrestricted arcs to parcel nodes in $P_e(h)$. Parcel node i has an arc of capacity a_i to the sink. A flow saturating every source-interface arc is exactly a feasible allocation x. This is the capacitated feasibility construction underlying Gale's flow theorem (Gale 1957).

By max-flow/min-cut, saturation fails only if some interface set F has cost greater than the capacity of its payer set $N_h(F) = \bigcup_{e \in F} P_e(h)$. Group the contiguous components of F whose expanded payer neighbourhoods overlap. For each group, the union of payer sets equals the expanded neighbourhood of the group's interval hull. The hull may contain additional interfaces with non-negative costs. Applying Equation 6 to that hull bounds the group's original costs by its payer capacity. Different groups have disjoint payer sets, so summing gives the required bound for F. Saturation follows.

For the strict-equilibrium addendum, necessity requires every $a_i > 0$ and every block inequality to be strict. For sufficiency, choose a common $\varepsilon > 0$ smaller than every a_i and small enough that all finitely many block inequalities remain satisfied after replacing each capacity by $a_i - \varepsilon$. Proposition 2 supplies an allocation with load at most $a_i - \varepsilon < a_i$ for each parcel, so every parcel strictly prefers to remain high when all others are high.

Proof of Proposition 3

Feasibility is monotone in h because payer sets expand. At $h = N - 2$, every interface can be charged to every parcel. Proposition 2 then reduces to non-negative parcel margin and the full-boundary condition $\sum_e \kappa_e \le \sum_i a_i$; that condition also covers every smaller interface block. These conditions are necessary at every radius, so they are necessary and sufficient for h^* to exist.

Proof of Proposition 4

Fix x and let $r_i = c_i + \sum_e x_{ie} - b_i$. Full completion requires $t_i \ge r_i$ for every parcel. Summing and using $\sum_i t_i = f$ gives necessity. If Equation 9 holds, let $S = \sum_i b_i + f - \sum_i c_i - \sum_e \kappa_e \ge 0$. Choose non-negative z_i with $\sum_i z_i = S$ and set $t_i = r_i + z_i$. Then transfers sum to f and every completion payoff is non-negative. Minimising the required $f \ge 0$ gives Equation 10.

Appendix B: Numerical verification

The script compares the interval conditions with independent max-flow and full Hall-subset implementations. It checks all 22,140 endpoint parameter configurations with two to five parcels and integer margins and costs in $\{0, 1, 2\}$. Repeating the comparison at every admissible radius gives 85,806 radius-specific configurations; the implementations agree in every case.

The script also checks 5,000 seeded rational cases through 11 parcels and verifies the block-deficit witnesses. Separate assertions cover a budget-balanced transfer vector, completion payoffs, private-versus-social surplus accounting, outside finance, and the valuation-horizon comparative static.

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